Gas Turbine Cycle

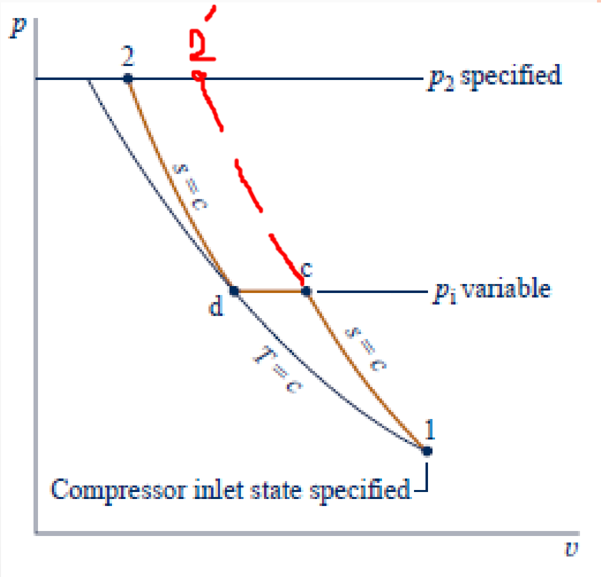
Lecture 7
Enhancements of gas cycle

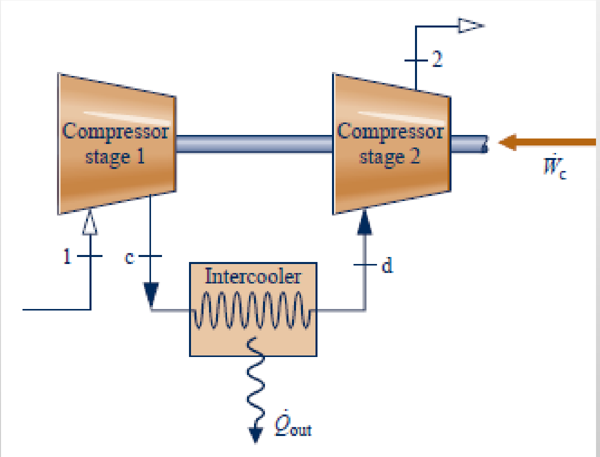
Tikrit university\ engineering college\ mechanical dept.

Intercooler pressure for minimum compressor work:

- If the inlet state and the exit pressure are specified for two – stage compressor operating at steady state, the minimum total work input occurs where the pressure ratio is the same across each stage.
- Assuming each compression process is isentropic, no pressure drop through the intercooler and the temp. at the inlet to each compressor stage is the same, kinetic and potential energy are neglected.
- The total compressor work input per unit mass flow is:

$$w_C = (h_C - h_1) + (h_2 - h_d) \ldots\ldots\ldots(1)$$





Intercooler pressure for minimum compressor work:

Since C_p is constant;

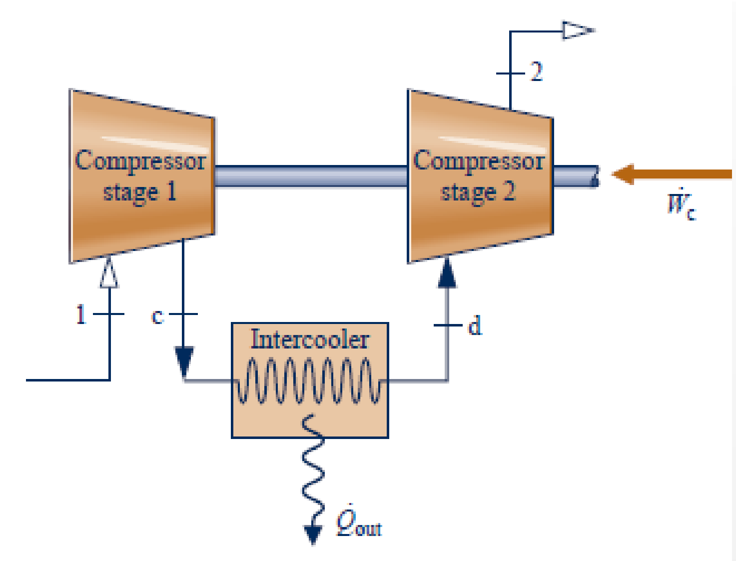
$$W_c = C_p\,(T_c - T_1) + C_p\,(T_2 - T_d) \quad \text{..........(2)}$$

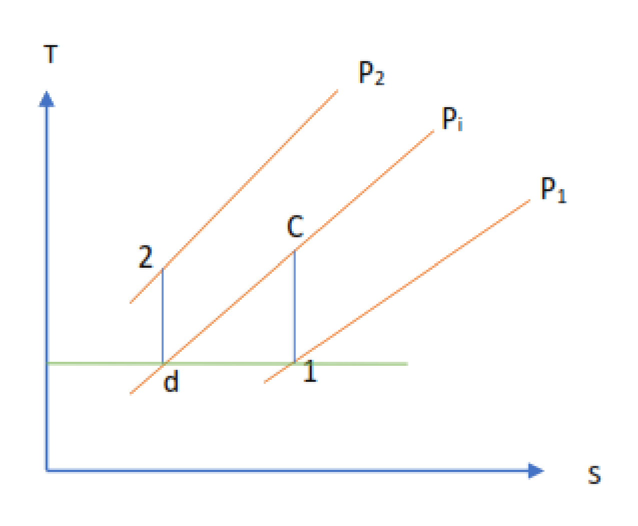
assuming $T_1 = T_d$:

$$w_c = C_p\,T_1\left[\frac{T_c}{T_1} + \frac{T_2}{T_1} - 2\right] \quad \text{.................(3)}$$

since the compression processes are isentropic and the specific heat ratio (k) is constant , the pressure & temp ratios across the compressor stage are related.

$$\frac{T_c}{T_1} = \left(\frac{P_i}{P_1}\right)^{\frac{k-1}{k}} \quad \& \quad \frac{T_2}{T_d} = \left(\frac{P_2}{P_i}\right)^{\frac{k-1}{k}} \quad \text{.............(4)}$$





Intercooler pressure for minimum compressor work:

With $T_1 = T_d$, substitute eq 4 in 3, we get:

$$W_c = C_p\, T_1 \left[\left(\frac{P_i}{P_1}\right)^{\frac{k-1}{k}} + \left(\frac{P_2}{P_i}\right)^{\frac{k-1}{k}} - 2 \right] \text{.............................(5)}$$

- Hence, for specified values of T_1, P_1, P_2 and C_P, the value of the total compressor work input varies with the intercooler pressure only.

- To determine the pressure P_i that minimize the total compressor work, form the derivative:

$$\frac{\partial W_c}{\partial P_i} = \frac{\partial}{\partial p_i}\left[C_p\, T_1 \left[\left(\frac{P_i}{P_1}\right)^{\frac{k-1}{k}} + \left(\frac{P_2}{P_i}\right)^{\frac{k-1}{k}} - 2 \right]\right] \text{............(6)}$$

$$= C_p\, T_1 \left(\frac{k-1}{k}\right) \left[\left(\frac{P_i}{P_1}\right)^{\frac{-1}{k}} \left(\frac{1}{P_1}\right) + \left(\frac{P_2}{P_i}\right)^{\frac{-1}{k}} \left(-\frac{P_2}{P_i^2}\right) \right]$$

$$= C_p\, T_1 \left(\frac{k-1}{k}\right)\left(\frac{1}{P_i}\right) \left[\left(\frac{P_i}{P_1}\right)^{\frac{k-1}{k}} - \left(\frac{P_2}{P_i}\right)^{\frac{k-1}{k}} \right]$$

Intercooler pressure for minimum compressor work:

When the partial derivative is set to zero, with constant T_1 , P_1 , P_2 & Cp, the desired relationship is obtained:

$$\left[\left(\frac{P_i}{P_1}\right)^{\frac{k-1}{k}} - \left(\frac{P_2}{P_i}\right)^{\frac{k-1}{k}}\right] = 0$$

or $\left[\left(\frac{P_i}{P_1}\right)^{\frac{k-1}{k}} = \left(\frac{P_2}{P_i}\right)^{\frac{k-1}{k}}\right]$

And $\frac{p_i}{p_1} = \frac{p_2}{p_i} \longrightarrow P_i^2 = P_1 P_2$

$P_i = \sqrt{P_1 P_2}$